A Comparison of a CA and a Macroscopic Model

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Introduction

In this paper we describe a relation between a microscopic stochastic traffic cellular automaton model (i.e., the STCA model [NS92]) and the macroscopic first order continuum model (i.e., the LWR model [LW55, Ric56]). Already, relations between both types of models have been investigated (e.g., [Nag96]). Our approach is however different, in that it provides a practical methodology for specifying the fundamental diagram to the LWR model, assuming that a stationarity condition holds on the STCA's rules. The innovative aspect is that we can incorporate the STCA's stochasticity explicitly in the construction of the fundamental diagram used by the LWR model.

Background

A first type of traffic flow models uses a fluid dynamics approach, in which the collective behaviour of infinitesimally small particles is described, using aggregate quantities such as flow q, density k and (space) mean speed v. This type of models can be computed using cell-based numerical schemes (e.g., using the Godunov scheme [Dag95, Leb96]). Later, microscopic traffic flow models have been developed that explicitly describe vehicle interactions at a high level of detail. During the early nineties, these models were reconsidered from an angle of particle physics: cellular automata models were applied to traffic flow theory, resulting in fast and efficient modelling techniques for microscopic traffic flow models [NS92]. These cellular automata models can be looked upon as a particle based discretisation scheme for macroscopic traffic flow models. It is from this latter point of view that our paper addresses the common structure between the seminal STCA model and the first order LWR model.

Methodology

We assume that we have the ruleset of the STCA available, as well as the maximum allowed speed $v_{\max}$ and the noise term p. Furthermore, a discretisation is assumed, expressed in the cell length ΔX, the time step ΔT, and its coupled speed increment $\Delta V = \Delta X / \Delta T$.

Relating both models is done using a simple two-step approach:

(1) Rewrite the STCA's rules, assuming a stationarity condition holds.

(2) Convert these new rules into a distance-gap/speed diagram, that is equivalent to a stationary fundamental diagram.

Starting from the ruleset of the STCA, we rewrite it using a min-max formulation. Instead of having several individual rules that give a discrete speed, we now have one rule that returns a continuous speed. The stationarity condition mentioned in (2), asserts that the speed *v(t)* of a vehicle at time *t* is the same as its speed at time *(t + ΔT)*. This allows us to reformulate the new min-max rule as a set of linear inequalities that express constraints on the relations between the speed *v(t)* of a vehicle, v_{max}, *p* and the vehicle's distance gap *h(t)*.

These inequalities together form a set of boundaries that can be plotted in a diagram that shows the distance gap of a vehicle versus its speed (see Figure 1 for an example). This diagram is equivalent to a stationary (*k*,*q*) fundamental diagram, which is then specified as a parameter to the LWR model.

An illustrative case study

We apply our methodology to a case study, in which we model a road that has a middle part with a reduced maximal allowed speed (e.g., an elevation, or a speed limit, …).

We simulate this road using on the hand the STCA (assuming a certain noise level *p*), and on the other hand the LWR model (both analytical and numerical).
We conclude with a comparison of both models based on their spatio-temporal behavior.

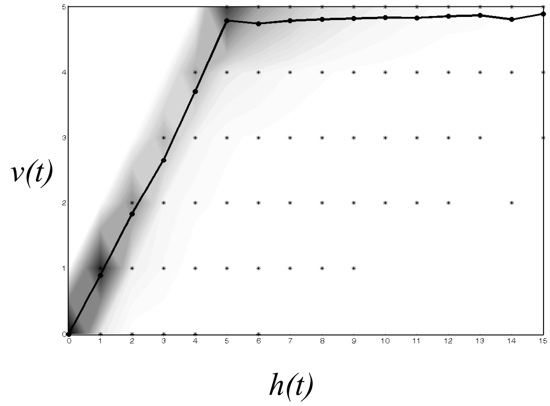

Figure 1: distance gap *h(t)* of a vehicle versus its speed *v(t)*. The average speed is shown as a solid line.

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